Name: Period: Date:

Math Journal: Dueling and The Secret Cubic Equation

In the late 15th and early 16th centuries, mathematicians challenged one another to public contests. Wealthy people placed bets on who would win and the mathematician who won the duel would not only win prize money, he would also get publicity that might result in a job offer from a university. When one mathematician challenged another to this type of duel, they each posed a set of math problems to his opponent. They exchanged problems and worked on them for a set amount of time. Whoever got the most correct in that time was declared winner. Since a lot of money and prestige was at stake, mathematicians worked night and day to develop formulas and short-cuts to solve time consuming problems. When they discovered one, they kept it secret all of their life because it gave them an advantage in duels. The most famous duel involved solving cubic equations.

Italian mathematician Scipione del Ferro (1465–1526) was the first to develop a formula for solving cubic equations, but it only worked for those in the form $x^3 + mx = n$. He kept his formula secret until he told his student, Antonio Fiore, the formula on his death bed in 1526.

In 1530, Niccolo Tartaglia was challenged by Fiore, who thought he could best him using the formula he learned from his teacher. Tartaglia had been working on his own formula for years and had perfected it but kept it secret. Tartaglia won the contest, the money, and a job lecturing at a university after besting Fiore.

Gerolamo Cardano was a doctor by trade who also dabbled in mathematics. He made most of his money gambling and writing books about using mathematics to cheat at gambling. Cardano struck up a friendship with Tartaglia and somehow coaxed him into revealing his secret for solving cubic equations. In 1539, Tartaglia told him the secret after making Cardano swear an oath that he would never share it with anyone. Tartaglia gave Cardano his formula in a poem:

When the cube and things together
Are equal to some discreet number,
Find two other numbers differing in this one.
Then you will keep this as a habit
That their product should always be equal
Exactly to the cube of a third of the things.
The remainder then as a general rule
Of their cube roots subtracted
Will be equal to your principal thing
In the second of these acts,
When the cube remains alone,
You will observe these other agreements:
You will at once divide the number into two parts
So that the one times the other produces clearly
The cube of the third of the things exactly.
Then of these two parts, as a habitual rule,
You will take the cube roots added together,
And this sum will be your thought.
The third of these calculations of ours
Is solved with the second if you take good care,
As in their nature they are almost matched.
These things I found, and not with sluggish steps,
In the year one thousand five hundred, four and thirty.
With foundations strong and sturdy
In the city girdled by the sea.

Six years later, Cardano learned about Scipione del Ferro's earlier formula and decided that publishing the formula would not be breaking his oath to Tartaglia, but would make him a lot of money. So in 1545, he published Ars Magna and gave credit to both Scipione del Ferro and Tartaglia for coming up with the formula independently.

Here is the formula that Cardano published for cubics given in the form $x^3 + ax + b = 0$.

$$Let\ R = \left(\frac{1}{2}b\right)^2 + \frac{a^3}{27}$$

$$Then, x = \left[-\frac{1}{2}b + \sqrt{R}\right]^{1/3} + \left[-\frac{1}{2}b - \sqrt{R}\right]^{1/3}$$

Tartaglia took Cardano to court several times to sue for the income he lost from his formula being made public. He never won in court. So Tartaglia challenged Cardano to another public math contest to try to win his lost income back from Cardano, but Cardano refused to participate. The challenge was eventually accepted by one of Cardano's students, Lodovico Ferrari, who bested Tartaglia. As a result Tartaglia not only went bankrupt, but lost his prestige and his job at the university where he was working. To get revenge, Tartaglia bribed Cardano's son to testify against his father in 1570 when Cardano was put on trial for heresy for calculating and publishing the horoscope of Jesus Christ in 1554. Cardano was convicted, sentenced to several months in prison, and lost his job as a university professor. Tartaglia got his revenge, but because Cardano was the first to publish the formula, he is the one given credit in most textbooks for it.

Use Cardano's method to find the real root of each cubic equation. Show all work. Round answers to three decimal places.

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1. $x^3 + 8x + 3 = 0$

2. $x^3 - 2x - 5 = 0$

3. $x^3 + 4x - 1 = 0$

4. $x^3 - x + 2 = 0$