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## Math Journal: Compound Interest and the Number $e$

Jacob Bernoulli (1654-1705) discovered the constant  $e$  when studying problems involving compound interest.

The example he used was the most basic one he could think of... a savings account that starts with \$1.00 and pays 100 percent interest per year. He calculated the interest for each of the following types of compounding.

**1] Calculate the value of the account after 1 year when interest is compounded in the following ways.**

**DO NOT ROUND THE DECIMALS SHOWN BY THE CALCULATOR! The first six have been done for you.**

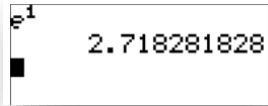
Frequency of compounding	Number of compoundings per year	$A = P \left(1 + \frac{r}{n}\right)^{nt}$ where $P$ is the starting amount, $r$ is the interest rate as a decimal, $n$ is the number of compoundings per year, $t$ is time (in years), and $A$ is the amount in the savings account after $t$ years.	Amount in savings account after 1 year
annually	1	$1 \left(1 + \frac{1}{1}\right)^{1(1)} = \left(\frac{2}{1}\right)^1 = 2$	\$2.00000000
semiannually	2	$1 \left(1 + \frac{1}{2}\right)^{2(1)} = \left(\frac{3}{2}\right)^2 = 2.25$	\$2.25000000
quarterly	4	$1 \left(1 + \frac{1}{4}\right)^{4(1)} = \left(\frac{5}{4}\right)^4 \approx 2.44140625$	\$2.44140625
monthly	12	$1 \left(1 + \frac{1}{12}\right)^{12(1)} = \left(\frac{13}{12}\right)^{12} \approx 2.61303529$	\$2.61303529
weekly	52	$1 \left(1 + \frac{1}{52}\right)^{52(1)} = \left(\frac{53}{52}\right)^{52} \approx 2.692596954$	\$2.692596954
daily	365	$1 \left(1 + \frac{1}{365}\right)^{365(1)} = \left(\frac{366}{365}\right)^{365} \approx 2.714567482$	\$2.714567482
hourly			
every minute			

Bernoulli noticed that this sequence approaches a limit (the force of interest) for more and smaller compounding intervals. Compounding weekly yields \$2.692597..., while compounding daily yields \$2.714567..., just two cents more.

**2] How much more interest is earned from compounding every minute versus every hour?**

Using  $n$  as the number of compounding intervals, with interest of 100% of  $n$  in each interval, the limit (maximum value) for  $n$  getting infinitely larger is the number that came to be known as  $e$ .  $e$  is irrational, just like  $\pi$ , which means it is decimal that never repeats and never ends.

Your calculator approximates  $e$  to 9 decimal places as



More generally, an account that starts at \$1, and yields  $A = 1 \left(1 + \frac{r}{n}\right)^{nt}$  dollars with an interest rate of  $r$ , will yield  $A = e^r$  dollars with **continuous compounding**, which is infinitely many compoundings per year.

Bernoulli later proved mathematically that:  $e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$  which will make more sense when you study calculus.

Bernoulli used the most basic case to develop the idea of continuous compounding... when the account starts at \$1 and earns \$100% interest. He then generalized the formula for different starting amounts, interest rates, and durations this way...

$$A = P \left(1 + \frac{r}{n}\right)^{nt} \text{ will yield } A = Pe^{rt} \text{ when interest is compounded continuously.}$$

Even though Bernoulli was the first to discover the constant for continuous compounding, it was given the name  $e$  in honor of Leonard Euler (1707-1783), who was the first to publish it.

**Use Bernoulli's formula to solve the following problems. Show your work and give units for your answer.**

	<p>3] A loan shark lends a gambler \$1000 to cover a debt. He charges 35% annual interest compounded continuously. Because he can't pay, the gambler skips town for a while until he earns enough money to pay his debt. How much does the gambler owe the loan shark when he returns 2 years later?</p>	<p>4] If you invest 1 penny in an account earning 1% interest compounded continuously, how much will the account be worth in 2000 years?</p>	
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**$e$  can be used in applications other than just calculating interest. Solve the following. Show work and give units.**

	<p>5] Radioactive iodine is used by doctors as a tracer in diagnosing certain thyroid gland disorders. This type of iodine decays in such a way that the mass remaining after <math>t</math> days is given by the function <math>m(t) = 6e^{-0.087t}</math> where <math>m(t)</math> is measured in grams. Find the size of the mass that remains after 45 days, rounded to two decimal places.</p>	<p>6] For a certain type of tree, the diameter <math>D</math> (in feet) depends on the tree's age <math>t</math> (in years) according to the model <math>D(t) = \frac{5.4}{1+2.9e^{-0.01t}}</math>. Find the diameter of a 20 year old tree, rounded to two decimal places.</p>	
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